

$\rho - \omega$ -Interference in J/ψ -Decays and $\rho \rightarrow \pi^+\pi^-\pi^0$ Decay *

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Abstract We study $\rho - \omega$ -interference by analyzing $J/\psi \rightarrow \pi^+\pi^-\pi^0\pi^0$. PDG-2002 data on J/ψ decays into PP and PV (P denotes pseudoscalar mesons; V , vector mesons) are used to fit a generic model which describes the J/ψ decays. From the fits, we obtain anomalously large branching ratio $Br(\rho^0 \rightarrow \pi^+\pi^-\pi^0) \sim 10^{-3} - 10^{-2}$. A theoretical analysis for it is also provided, and the prediction is in good agreement with the anomalously large $Br(\rho^0 \rightarrow \pi^+\pi^-\pi^0)$. By the fit, we also get the $\eta - \eta'$ -mixing angle $\theta = -19.68^\circ \pm 1.49^\circ$ and the constituent quark mass ratio $m_u/m_s \sim 0.6$ which are all reasonable.

Keyword J/ψ -decays, $\rho - \omega$ interference, SU(3)-breaking effect, $\eta - \eta'$ mixing

1 introduction

Recently it has been predicted in ref.^[1] that there is a large isospin symmetry breaking enhancement effect in the decay $\rho^0 \rightarrow \pi^0\gamma$ comparing with $\rho^\pm \rightarrow \pi^\pm\gamma$ due to $\rho - \omega$ -interference, which was called as hidden isospin-breaking effects in ^[1]. This prediction has been confirmed by the renewed data in PDG-2002^[2]. Following the discussion of ^[1], it could be expected that a similar large hidden isospin-breaking effect should also exist in $\rho^0 \rightarrow 3\pi$ due to $\rho - \omega$ interference. In this paper we try to analyze $J/\psi \rightarrow (\rho^0, \omega)(\pi^0, \eta, \eta') \rightarrow (3\pi)(\pi^0, \eta, \eta')$, and to reveal $\rho - \omega$ -interference effect and then finally to abstract out the branching ratio of $\rho^0 \rightarrow 3\pi$. Such an analysis should be necessary for further confirming the enhancement effects mentioned above, and be also interesting for the G -parity violating process studies.

It is very difficult to directly measure $B_r(\rho^0 \rightarrow 3\pi)$ experimentally both because $\Gamma_\rho \gg (m_\omega - m_\rho)$ and because the G -parity conserving decay mode of $\omega \rightarrow 3\pi$ is dominate. This is the reason why there is still no a reliable value for $B_r(\rho \rightarrow 3\pi)$ yet so far in the literature^{[2][3]}. Fortunately, this quantity can be obtained by fitting the data of $J/\psi \rightarrow PP$ and PV (where P

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denotes the pseudoscalar meson nonet (π, K, η, η') , and V , the vector mesons $(\rho, \omega, K^*, \phi)$ ^[4]. Actually, more than fifteen years ago, $Br(\rho^0 \rightarrow 3\pi)$ was estimated in ref.^[4] by using the MARK-III data of $(J/\psi \rightarrow PP, PV)$ -decays with $P = (\pi, K)$ and $V = (\rho, \omega, K^*)$. However, people including the authors of ref.^[4] does not think their result of $Br(\rho^0 \rightarrow 3\pi)$ is very reliable (see the discussion in ref.^[4] and ref.^[2]). The reasons are multi-ply, and some of them may be as follows: 1, the J/ψ -decay data quality at that time was not good enough ; 2, the fitting is not complete because the processes of $(J/\psi \rightarrow \eta V, \eta' V, P\phi)$ were not considered; 3, lacking a theoretical understanding why their result is so significantly different from the result of ref.^[3] which is quoted by PDG^[2]. Our motive of this paper is to try to solve those problems: 1, We shall use nowadays data of $(J/\psi \rightarrow PP, PV)$ in PDG-2002^[2] to perform the fit; 2, $(J/\psi \rightarrow \eta V, \eta' V, P\phi)$ will be considered in our analysis; 3, And the rationality of the result will be argued, i.e., we'll see that the result is just consistent with the theoretical analysis in ref.^[1].

The contents of the paper are organized as follows. In section 2 we describe the $\rho - \omega$ -interference in the process $J/\psi \rightarrow \pi^+\pi^-\pi^0\pi^0$, and give the the branching ratio formulas for $(J\psi \rightarrow PP, PV)$. In section 3, by using the PDG-2002 J/ψ decay data and the formulas given in the section 2 we perform the datum fits. The $Br(\rho^0 \rightarrow \pi^+\pi^-\pi^0)$ is obtained. The section 4 is devoted to estimate $Br(\rho^0 \rightarrow \pi^+\pi^-\pi^0)$ by theoretical analysis. Finally, we briefly discuss the results.

2 $\rho - \omega$ Interference and branching ratios of J/ψ decays into PV and PP

The elusive $\rho^0 \rightarrow \pi^+\pi^-\pi^0$ decay could be observed in J/ψ decays into the $\pi^+\pi^-\pi^0\pi^0$ final state^[4]. Indeed, this decay can proceed through the interfering channels $J/\psi \rightarrow \omega\pi^0 \rightarrow \pi^+\pi^-\pi^0\pi^0$ and $J/\psi \rightarrow \rho\pi^0 \rightarrow \pi^+\pi^-\pi^0\pi^0$. Because $J/\psi \rightarrow \rho\pi^0$ is caused both by strong interaction via 3 gluons and by electromagnetic (EM) interaction and $J/\psi \rightarrow \omega\pi^0$ is caused by EM interaction merely, the passibility of the decay $J/\psi \rightarrow \rho\pi^0$ is much larger than one of $J/\psi \rightarrow \omega\pi^0$, i.e., $\Gamma(J/\psi \rightarrow \rho\pi^0) \gg \Gamma(J/\psi \rightarrow \omega\pi^0)$ (by using the 2002-PDG data^[2] we have $\Gamma(J/\psi \rightarrow \rho\pi^0) \simeq 2.5 \times 10^2 \Gamma(J/\psi \rightarrow \omega\pi^0)$). Consequently, even though $\Gamma(\omega \rightarrow \pi^+\pi^-\pi^0)$ may be much larger than $\Gamma(\rho \rightarrow \pi^+\pi^-\pi^0)$, it is still hopeful to measure $\Gamma(\rho \rightarrow \pi^+\pi^-\pi^0)$ by studying the $\rho - \omega$ -interference effects in the process of $J/\psi \rightarrow \pi^+\pi^-\pi^0\pi^0$.

To $J/\psi \rightarrow (\rho, \omega)\pi^0 \rightarrow 3\pi\pi^0$, the corresponding s-dependence Breit-Wigner is written as^[4] $F(s) \equiv BW_\omega(s) + \epsilon e^{i\theta'} BW_\rho(s)$, where

$$BW_i = \sqrt{\frac{m_i \Gamma_i}{\pi}} \frac{1}{m_i^2 - s - im_i \Gamma_i} \quad (1)$$

is the normalized Breit-Wigner curve for $i = \omega, \rho$ resonance with the mass of m_i and total width of Γ_i . The factor $\epsilon(\theta')$ is the modulus(phase) of the amplitude proceeding through the ρ resonance relative to ω resonance, which can be written as:

$$\epsilon = \frac{|A(J/\psi \rightarrow \rho^0\pi^0)|}{|A(J/\psi \rightarrow \omega\pi^0)|} \times \sqrt{\frac{BR(\rho^0 \rightarrow 3\pi)}{BR(\omega \rightarrow 3\pi)}} \quad (2)$$

and $\theta' = \theta_{(J/\psi \rightarrow i\pi^0)} + \theta_{(i \rightarrow 3\pi)}$ with $i = \rho, \omega$.

As we have the relations $m_\rho \simeq m_\omega = m$ and $\Gamma_\rho \neq \Gamma_\omega$, the total effect is integrated over s ,

$$\int ds |F(s)|^2 = 1 + \epsilon^2 + 2\epsilon \cos \theta' \frac{2\sqrt{\Gamma_\omega \Gamma_\rho}}{\Gamma_\omega + \Gamma_\rho}. \quad (3)$$

The third term of the expression (3), $2\epsilon \cos \theta' (\frac{2\sqrt{\Gamma_\omega \Gamma_\rho}}{\Gamma_\omega + \Gamma_\rho})$, is the ρ - ω interference term. According to ref.^{[4]1}, the phase θ' is equal to zero, and the interference effect produces a magnificent factor as a whole (we assume that the ω resonance contribution is one), ie.

$$\int ds |F(s)|^2 = 1 + \epsilon^2 + \epsilon, \quad (4)$$

where $\Gamma_\rho \simeq 16\Gamma_\omega$ ^[2] has been used. Then, the interference between ρ and ω in the $J/\psi \rightarrow 4\pi$ provides a relation as follows

$$BR(J/\psi \rightarrow \omega(\rho)\pi^0 \rightarrow 4\pi) = (1 + \epsilon^2 + \epsilon)BR(J/\psi \rightarrow \omega\pi^0 \rightarrow 4\pi). \quad (5)$$

The $BR(J/\psi \rightarrow \omega(\rho)\pi^0 \rightarrow 4\pi)$ can be detected directly in experiments, but there is no a direct experiment way to measure $BR(J/\psi \rightarrow \omega\pi^0 \rightarrow 4\pi)$ in the right hand side of the above relation. Fortunately, it can be got by fitting the branch ratios of $J/\psi \rightarrow PP$ and PV (where P and V denote pseudoscalar- and vector mesons respectively)^[4]. As both $BR(J/\psi \rightarrow \omega(\rho)\pi^0 \rightarrow 4\pi)$ and $BR(J/\psi \rightarrow \omega\pi^0 \rightarrow 4\pi)$ are known, we will have ϵ by Eq. 5 and then obtain desired quantity $BR(\rho^0 \rightarrow 3\pi)$ via Eq. 2.

Decays of J/ψ into (PV) and into (PP) can be factorized by a very simple and general consideration described as follows^[4]. The decays proceed through a strongly interacting three-gluon (ggg) intermediate state and through electromagnetic interaction mediated by one photon γ and (γgg) states. In the gluonic case, there are two $I = 0$ transitions $c\bar{c} \rightarrow ggg \rightarrow \frac{(u\bar{u}+d\bar{d})}{\sqrt{2}}$ and $c\bar{c} \rightarrow ggg \rightarrow s\bar{s}$, which are proportional to the amplitude A and $\frac{\lambda A}{\sqrt{2}}$ respectively. The parameter λ accounts for flavor- $SU(3)$ breaking effect. The electromagnetic transitions generate the $I = 1$ state $\frac{(u\bar{u}-d\bar{d})}{\sqrt{2}}$ and two $I = 0$ states ($\frac{(u\bar{u}+d\bar{d})}{\sqrt{2}}$ and $s\bar{s}$). Their amplitude are proportional to $3a$, a and $-\sqrt{2}\lambda a$ respectively. The flavor space wave functions for P including η and η' read

$$P = \lambda^a \Phi^a = \sqrt{2} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{\eta_1}{\sqrt{3}} \end{pmatrix}, \quad (6)$$

where $\eta_8 = \eta \cos \theta + \eta' \sin \theta$ and $\eta_1 = \eta' \cos \theta - \eta \sin \theta$ with θ as $(\eta - \eta')$ -mixing angle, and the V -wave functions including ϕ 's read

$$V = \sqrt{2} \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \quad (7)$$

With the above, the decay amplitudes of $(J/\psi \rightarrow PP, PV)$ are as follows

$$A(\pi^+\pi^-) = 3a, \quad (8)$$

¹ $\theta = 0$ in $J/\psi \rightarrow (\rho \text{ or } \omega)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ and $\theta = \pi/2$ in $J/\psi \rightarrow (\rho \text{ or } \omega)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ in the paper of A.Bramon and J.Casulleras, Phys.Lett., 1986, **173B**: 97.

$$A(K^+K^-) = \frac{1}{2}(1-\lambda)A + (2+\lambda)a, \quad (9)$$

$$A(K^0\bar{K}^0) = \frac{1}{2}(1-\lambda)A - (1-\lambda)a, \quad (10)$$

$$A(\rho^0\pi^0) = f_v(A+a), \quad (11)$$

$$A(K^{*+}K^-) = f_v[\frac{1}{2}(1+\lambda)A + (2-\lambda)a], \quad (12)$$

$$A(K^{*0}\bar{K}^0) = f_v[\frac{1}{2}(1+\lambda)A - (1+\lambda)a], \quad (13)$$

$$A(\omega\pi^0) = f_v(3a), \quad (14)$$

$$A(\rho\eta') = f_v(3aX_{\eta'}), \quad (15)$$

$$A(\omega\eta') = f_v[(A+a)X_{\eta'} + \sqrt{2}rA(\sqrt{2}X_{\eta'} + Y_{\eta'})], \quad (16)$$

$$A(\rho\eta) = f_v(3aX_{\eta}), \quad (17)$$

$$A(\omega\eta) = f_v[(A+a)X_{\eta} + \sqrt{2}rA(\sqrt{2}X_{\eta} + Y_{\eta})], \quad (18)$$

$$A(\phi\eta) = f_v[(A-2a)\lambda Y_{\eta} + rA(\sqrt{2}X_{\eta} + Y_{\eta})\frac{(1+\lambda)}{2}], \quad (19)$$

$$A(\phi\eta') = f_v[(A-2a)\lambda Y_{\eta'} + rA(\sqrt{2}X_{\eta'} + Y_{\eta'})\frac{(1+\lambda)}{2}], \quad (20)$$

where $X_{\eta} = \sqrt{\frac{1}{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta$, $X_{\eta'} = \sqrt{\frac{1}{3}}\sin\theta + \sqrt{\frac{2}{3}}\cos\theta$, $Y_{\eta} = -X_{\eta'}$, $Y_{\eta'} = X_{\eta}$. The additional parameter r is the relative weight of the disconnected diagram to connected diagram for the decays involving the final state η or η' [5][6]. The Eqs. 8- 14 are same as ones in ref.[4], and others are new.

The corresponding branching ratios of these decays are following

$$Br(\pi^+\pi^-) = 9a^2, \quad (21)$$

$$Br(K^+K^-) = |\frac{1}{2}(1-\lambda)A + (2+\lambda)ae^{i\phi}|^2, \quad (22)$$

$$Br(K^0\bar{K}^0) = |\frac{1}{2}(1-\lambda)A - (1-\lambda)ae^{i\phi}|^2, \quad (23)$$

$$Br(\rho^0\pi^0) = f_v^2|(A+ae^{i\phi})|^2, \quad (24)$$

$$Br(K^{*+}K^-) = f_v^2|\frac{1}{2}(1+\lambda)A + (2-\lambda)ae^{i\phi}|^2, \quad (25)$$

$$Br(K^{*0}\bar{K}^0) = f_v^2|\frac{1}{2}(1+\lambda)A - (1+\lambda)ae^{i\phi}|^2, \quad (26)$$

$$Br(\omega(\rho^0)\pi^0) = (1+\epsilon+\epsilon^2)f_v^29a^2, \quad (27)$$

$$Br(\rho\eta') = f_v^2|3aX_{\eta'}|^2, \quad (28)$$

$$Br(\omega\eta') = f_v^2|(A+ae^{i\phi})X_{\eta'} + \sqrt{2}rA(\sqrt{2}X_{\eta'} + Y_{\eta'})|^2, \quad (29)$$

$$Br(\rho\eta) = f_v^2|3aX_{\eta}|^2, \quad (30)$$

$$Br(\omega\eta) = f_v^2|(A+ae^{i\phi})X_{\eta} + \sqrt{2}rA(\sqrt{2}X_{\eta} + Y_{\eta})|^2, \quad (31)$$

$$Br(\phi\eta) = f_v^2 |(A - 2ae^{i\phi})\lambda Y_\eta + rA(\sqrt{2}X_\eta + Y_\eta)\frac{1+\lambda}{2}|^2, \quad (32)$$

$$Br(\phi\eta') = f_v^2 |(A - 2ae^{i\phi})\lambda Y_{\eta'} + rA(\sqrt{2}X_{\eta'} + Y_{\eta'})\frac{1+\lambda}{2}|^2, \quad (33)$$

where ϕ is their relative phase between A and a , and the parameter A and a are real. In the Eq. 24 the $\rho - \omega$ interference effect is subtracted from the branching ratio of $J/\psi \rightarrow \rho\pi^0$. In the Eq. 27, a magnificent factor $1 + \epsilon + \epsilon^2$ has been added due to Eq. 5 in order to taking the $\rho - \omega$ interference effects into account. Actually, through directly detecting the data of $J/\psi \rightarrow 4\pi$ one can only get $Br(\omega(\rho^0)\pi^0)$ rather than $Br(\omega\pi^0)$ which is equal to $f_v^2 9a^2$.

In the branching ratio formulae of Eqs. 21- 33 we do not write out the corresponding phase-space factors explicitly which are proportional to the cube of the final momenta in two-body decays. They will be taken into account in the practical phenomenological fit later.

3 Datum fit to obtain $Br(\rho \rightarrow \pi^+\pi^-\pi^0)$

Let's now use the data in PDG-2002^[2] to perform the fit to all branching ratios of Eq. 21- 33. This fit will lead to determining the ϵ and $Br(\rho \rightarrow \pi^+\pi^-\pi^0)$. The experimental branching ratio data of PDG-2002 are listed in the second column of Table 1.

Firstly, following ref.^[4], we use the branch ratio data of $(J/\psi \rightarrow PP, V(\pi^0, K))$ only to perform a fit to Eqs. 21- 27 (call it as $(PP, V(\pi^0, K))$ -fit hereafter). In this case, there are seven equations with six adjustable free parameters a , A , λ , ϕ , f_V , ϵ , and hence it is an over-determination problem with potential of predictions. The fit with minimum $\chi^2 = 0.46$ leads to the values of the parameters and seven corresponding branching ratios listed in the third column of Table 1, in which $(\omega\pi^0)_{uncor}$ and $(\omega\pi^0)_{cor}$ represent $Br(J/\psi \rightarrow \omega(\rho)\pi^0 \rightarrow 4\pi)$ and $Br(J/\psi \rightarrow \omega\pi^0 \rightarrow 4\pi)$ respectively, i.e.,

$$Br(J/\psi \rightarrow \omega\pi^0)_{cor} = f_v^2 9a^2. \quad (34)$$

In the fit (see Table 1), we have $(\omega\pi^0)_{uncor} = (4.2 \pm 0.61) \times 10^{-4}$, $a = 0.21 \pm 0.02$, $A = 2.94 \pm 0.72$ and the interference factor $\epsilon = 0.71 \pm 0.58$, then we obtain $(\omega\pi^0)_{cor} = f_v^2 9a^2 = (1.89 \pm 0.83) \times 10^{-4}$. In other hand, from Eqs.(2)(11)(14), ϵ reads

$$\epsilon = \frac{|A + ae^{i\phi}|}{|3a|} \times \sqrt{\frac{Br(\rho \rightarrow 3\pi)}{Br(\omega \rightarrow 3\pi)}}. \quad (35)$$

Then the branching ratio of $\rho \rightarrow \pi^+\pi^-\pi^0$ is predicted as follows

$$Br(\rho \rightarrow 3\pi) = Br(\omega \rightarrow 3\pi) \times \left(\frac{3|a|}{|A + ae^{i\phi}|}\right)^2 \epsilon^2. \quad (36)$$

Substituting the a -, A - and ϕ -values obtained from the fit (see the third column of Table 1) and experiment data of $Br(\omega \rightarrow 3\pi)$ into Eq. 34 and Eq. 36, we then obtain the $(PP, V(\pi^0, K))$ -fit's results as follows

$$Br(J/\psi \rightarrow \omega\pi^0)_{cor}|_{(PP, V(\pi^0, K))} = (1.89 \pm 0.83) \times 10^{-4}, \quad (37)$$

$$Br(\rho \rightarrow \pi^+\pi^-\pi^0)|_{(PP, V(\pi^0, K))} = (2.0 \pm 1.64) \times 10^{-2}. \quad (38)$$

TABLE 1: The second column displayed the experimental values for the branch ratios of $J/\psi \rightarrow PP$ and $J/\psi \rightarrow PV$ in PDG-2000 datum. The results of a fit to the first seven branching ratios is listed in the third column. The results of a fit to the total thirteen branching ratios is listed in the forth column.

J/ψ decay	PDG-2002(10^{-4})	a partial fit1(10^{-4})	a global fit2(10^{-4})
1. $\pi^+\pi^-$	1.47 ± 0.23	1.44 ± 0.23	1.92 ± 0.05
2. K^+K^-	2.37 ± 0.31	2.45 ± 0.28	2.04 ± 0.08
3. $K^0\bar{K}^0$	1.08 ± 0.14	1.06 ± 0.14	0.87 ± 0.05
4. $\rho^0\pi^0$	42.0 ± 5	43.04 ± 4.48	41.97 ± 0.68
5. $K^{*+}K^-$	25.0 ± 2.0	24.11 ± 1.41	23.64 ± 0.5
6. $K^{*0}K^0$	21.0 ± 2.0	21.66 ± 1.7	24.21 ± 0.49
7. $(\omega\pi^0)_{uncor}$	4.2 ± 0.6	4.2 ± 0.61	4.2 ± 0.2
8. $(\rho\eta')$	1.05 ± 0.18		0.7 ± 0.05
9. $(\omega\eta')$	1.67 ± 0.25		1.73 ± 0.11
10. $(\rho\eta)$	1.93 ± 0.23		1.82 ± 0.08
11. $(\omega\eta)$	15.8 ± 1.6		18.32 ± 0.36
12. $(\phi\eta)$	6.5 ± 0.7		5.85 ± 0.23
13. $(\phi\eta')$	3.3 ± 0.4		2.55 ± 0.23
χ^2		0.46/1	21.4/5
EDM		$0.45E - 06$	$0.69E - 06$
fit a		0.21 ± 0.02	0.24 ± 0.012
A		2.94 ± 0.72	2.69 ± 0.17
λ		0.6 ± 0.1	0.62 ± 0.03
ϕ		1.37 ± 0.14	1.6 ± 0.11
f_v		1.26 ± 0.36	1.38 ± 0.1
ϵ		0.71 ± 0.58	0.3 ± 0.16
θ			-0.343 ± 0.026
r			-0.144 ± 0.001
$(\omega\pi^0)_{cor}$		$(1.89 \pm 0.83) \times 10^{-4}$	$(3.02 \pm 0.2) \times 10^{-4}$
$\rho \rightarrow 3\pi$		$(2.0 \pm 1.64) \times 10^{-2}$	$(0.59 \pm 0.315) \times 10^{-2}$

Secondly, we perform more complete datum fit in which the processes of $J/\psi \rightarrow V\eta$ and $J/\psi \rightarrow V\eta'$ are included. In this case, there are 13 equations (21-33) and eight free parameters: a , A , λ , ϕ , f_V , ϵ , θ , r . And hence it is an over-determination problem with more constraints, and will be called as $(PP, V(\pi^0, K, \eta, \eta'))$ -fit hereafter. The results are as follows

$$Br(J/\psi \rightarrow \omega\pi^0)_{cor}|_{(PP, V(\pi^0, K, \eta, \eta'))} = (3.02 \pm 0.2) \times 10^{-4}, \quad (39)$$

$$Br(\rho \rightarrow \pi^+\pi^-\pi^0)|_{(PP, V(\pi^0, K, \eta, \eta'))} = (0.59 \pm 0.315) \times 10^{-2}, \quad (40)$$

$$\theta = -0.343 \pm 0.026 = -19.68^\circ \pm 1.49^\circ, \quad (41)$$

where η - η' -mixing angle θ is agreement with one in ref.^{[5][6]}, and both $Br(J/\psi \rightarrow \omega\pi^0)_{cor}$ and $Br(\rho \rightarrow \pi^+\pi^-\pi^0)$ are reasonable agreement with the results (37)(38) obtained by $(PP, V(\pi^0, K))$ -fit within the errors.

The parameter λ is the constituent quark mass ratio m_u/m_s which should be about 0.6^{[6][7][8]} due to light flavor SU(3)-breaking. The results of $\lambda \simeq 0.6 \pm 0.1$ for $(PP, V(\pi^0, K))$ -fit and $\lambda \simeq 0.62 \pm 0.03$ for $(PP, V(\pi^0, K, \eta, \eta'))$ -fit indicate the fits meet this requirement, and, hence, the results yielded by them are rather reliable.

4 Large isospin breaking effect in decay $\rho^0 \rightarrow \pi^+\pi^-\pi^0$

In this section, following ref.^[1], we provide a theoretical estimation to $Br(\rho^0 \rightarrow \pi^+\pi^-\pi^0)$. Using Feynman propagators method, the on-shell amplitude^[1] of the decay $\rho \rightarrow \pi^+\pi^-\pi^0$ is determined by

$$\mathcal{M}_{\rho^0 \rightarrow 3\pi} = \left(f_{\rho 3\pi} + \frac{\Pi_{\rho\omega}(p^2)f_{\omega 3\pi}}{p^2 - m_\omega^2 + im_\omega\Gamma_\omega} \right) \Big|_{p^2=m_\rho^2}, \quad (42)$$

where, the momentum-dependent $\rho^0 - \omega$ interference amplitude $\Pi_{\rho\omega}(q^2)$ is defined by the $\rho - \omega$ interaction Lagrangian $\mathcal{L}_{\rho\omega}$ as follows

$$\mathcal{L}_{\rho\omega} = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot x} \Pi_{\rho\omega}(p^2) (g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}) \omega_\mu(p) \rho_\nu^0(x). \quad (43)$$

The first and second term of expression (42) correspond to the contributions of direct coupling ($\rho^0 - 3\pi$) and ω resonance exchange respectively. Because $m_\rho \simeq m_\omega$ and Γ_ω is small, the denominator of the second term is small. Therefore, contribution from ω resonance exchange is large. This is called "hidden isospin symmetry breaking effect" according to ^[1]. This effect brings a significant contribution and plays an essential role in the decay $\rho \rightarrow \pi^+\pi^-\pi^0$. So when we deal with the decay of $\rho \rightarrow \pi^+\pi^-\pi^0$, the process $\rho \rightarrow \omega \rightarrow \pi^+\pi^-\pi^0$ must be considered.

In fact, the contributions of $\rho^0 - \omega$ interference are dominant and the direct coupling can be omitted. The direct coupling $f_{\rho 3\pi} \propto (m_d - m_u)$, therefore, it is very small. In order to be sure of this point, we derive this quantity in a practical model called as $U(2)_L \times U(2)_R$ chiral theory of mesons ^[9] in follows. Denoting the direct vertices of $\rho^0 - 3\pi$ as $\mathcal{L}_{\rho 3\pi} = f_{\rho 3\pi} \epsilon^{\mu\nu\alpha\beta} \epsilon_{ijk} \rho_\mu^0 \partial_\nu \pi^i \partial_\alpha \pi^j \partial_\beta \pi^k$, then $f_{\rho 3\pi}$ can be calculated in this theory^[9] and has the form

$$f_{\rho 3\pi} = -\frac{m_d - m_u}{\pi^2 g f_\pi^3 m} \left(1 - \frac{16c}{3g} + \frac{6c^2}{g^2} - \frac{8c^3}{3g^3} \right) \sim -2 \times 10^{-11} \text{MeV}^{-3}, \quad (44)$$

where the values of model's parameters m , g , c determined in ref.^[9] have been used. To the second term in the parentheses of expression (42), $\Pi_{\rho\omega}(m_\rho^2)$ has been determined to approximate $-4 \times 10^3 \text{MeV}^2$ ^[10, 11]. $\omega \rightarrow 3\pi$ is the dominant channel for ω -decays, and hence $f_{\omega 3\pi}$ can be estimated by using the width $\Gamma_{\omega \rightarrow 3\pi} = 7.5 \text{MeV}$. Its approximate value is about $3 \times 10^{-7} \text{MeV}^{-3}$. Thus the typical value of the second term in expression (42) is $(5 + 2i) \times 10^{-8} \text{MeV}^{-3}$ approximately. Comparing it with expression (44), we can see that the direct coupling $f_{\rho 3\pi}$ is indeed very small, and it is ignorable. Therefore, discarding $f_{\rho 3\pi}$ in expression (42), we have, approximately,

$$\Gamma_{\rho^0 \rightarrow 3\pi} = \left| \frac{\Pi_{\rho\omega}(m_\rho^2)}{m_\rho^2 - m_\omega^2 + im_\omega\Gamma_\omega} \right|^2 \Gamma_{\omega \rightarrow 3\pi}. \quad (45)$$

This equation means that the contributions due to $\rho - \omega$ -interference to $Br(\rho^0 \rightarrow 3\pi)$ are dominate, or the hidden isospin-breaking effects introduced in ^[1] are dominate for the process $\rho^0 \rightarrow 3\pi$. From expression (45) we obtain desired result as follows

$$BR(\rho^0 \rightarrow 3\pi) \simeq 0.2 \times 10^{-2}. \quad (46)$$

Our experiment datum fitting result (40) is consistent with this theoretical estimation result. This fact indicates that both $(PP, V(\pi^0, K))$ -fit and $(PP, V(\pi^0, K, \eta, \eta'))$ -fit are reasonable even though the resulting $BR(\rho^0 \rightarrow 3\pi)$ is much larger than one in ref.^[3] and rather closes the upper limit for it in ref.^[12].

5 Discussion

Through the study presented in the above, we conclude that ρ - ω -interference effects can be detected in the $J/\psi \rightarrow \pi^+\pi^-\pi^0\pi^0$ decay, which receives a contribution from the $\rho^0 \rightarrow \pi^+\pi^-\pi^0$ decay mode. J/ψ decays offer an almost unique opportunity for observing $\rho^0 \rightarrow \pi^+\pi^-\pi^0$, where the smallness of $Br(\rho^0 \rightarrow \pi^+\pi^-\pi^0)/Br(\omega \rightarrow \pi^+\pi^-\pi^0)$ is compensated by the large ratio $A(J/\psi \rightarrow \rho\pi^0)/A(J/\psi \rightarrow \omega\pi^0)$ between a (simply Zweig-forbidden) strong amplitude over an EM one. This is the key point for the practical determining $Br(\rho^0 \rightarrow \pi^+\pi^-\pi^0)$ through employing J/ψ decay branching ratios. Our results for 2 datum-fits are $Br(\rho \rightarrow \pi^+\pi^-\pi^0)|_{(PP,V(\pi^0,K))} = (2.0 \pm 1.64) \times 10^{-2}$ and $Br(\rho \rightarrow \pi^+\pi^-\pi^0)|_{(PP,V(\pi^0,K,\eta,\eta'))} = (0.59 \pm 0.315) \times 10^{-2}$ respectively, which are anomalously large and match each other within the errors.

In order to pursue whether these anomalously large results of $Br(\rho \rightarrow \pi^+\pi^-\pi^0)$ are reasonable or not, a theoretical estimation for ρ - ω -interference effects to the process of $(\rho \rightarrow \pi^+\pi^-\pi^0)$ has also been discussed in this paper. Following ref.^[1], we found that the contributions due to so called hidden isospin-breaking effects are dominate for the process $\rho \rightarrow \pi^+\pi^-\pi^0$. The theoretical prediction is $Br(\rho^0 \rightarrow 3\pi) \simeq 0.2 \times 10^{-2}$ which is in good agreement with our datum-fit results. Then, considering this fact and noting that both result of η - η' -angle θ and the result of constituent quark ratio $\lambda = m_u/m_s$ obtained by the fits are also reasonable, we conclude that $Br(\rho \rightarrow \pi^+\pi^-\pi^0) \sim 10^{-3} - 10^{-2}$ is reliable.

Finally, we like to argue that in order to reduce the error-bar of $Br(\rho \rightarrow \pi^+\pi^-\pi^0)$, more precisely experimental measurements to $(J/\psi \rightarrow PP, PV)$ are expected. The high quality data for J/ψ in the future BESIII would be useful.

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